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CRT-773

**Vehicle Routing with Stochastic Demands
and Restricted Failures**

by

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Centre de recherche sur les transports — Publication #773

May 1991— Revised November 1992

Dépôt légal – Bibliothèque nationale du Québec,
Bibliothèque nationale du Canada, 1992

RÉSUMÉ

Dans cet article, on étudie une classe de problèmes de tournées de véhicules (PTV) pour lesquels la demande des clients est stochastique. L'approche utilisée consiste à appliquer à ce type de problème des méthodes de résolution originellement conçues pour le cas déterministe. Dans le cas de problèmes avec contraintes probabilistes, on obtient une solution directement par une modification simple d'algorithmes conçus pour le PTV déterministe. Dans le cas de problèmes avec recours, on considère un certain nombre de scénarios avec nombre limité de pannes. La procédure consiste alors à résoudre une série de problèmes du voyageur de commerce (PVC). Le nombre de PVC résolus dépend alors du scénario considéré : il est plus élevé dans le cas de politiques de recours générales, et davantage restreint dans le cas de politiques plus restrictives.

Mots-clés : problèmes stochastiques de construction de tournées de véhicules, recours, problème du voyageur de commerce.

ABSTRACT

This paper considers a class of stochastic vehicle routing problems (SVRPs) with random demands, in which the number of potential failures per route is restricted either by the data or the problem constraints. These are realistic cases as it makes little sense to plan vehicle routes that systematically fail a large number of times. First, a chance constrained version of the problem is considered which can be solved to optimality by algorithms similar to those developed for the deterministic vehicle routing problem (VRP). Three classes of SVRP with recourse are then analyzed. In all cases, route failures can only occur at one of the last k customers of the planned route. Since in general, SVRPs are considerably more intractable than the deterministic VRPs, it is interesting to note that these realistic stochastic problems can be solved as a sequence of deterministic traveling salesman problems (TSPs). In particular, when $k = 1$ the SVRP with recourse reduces to a single TSP.

Key words : Vehicle routing, traveling salesman, chance constrained programming, stochastic recourse.

Introduction

The deterministic *Vehicle Routing Problem* (VRP) can be defined as follows. Let $G = (V, A)$ be a complete graph where $V = \{0, 1, \dots, n\}$ is a set of vertices representing cities or customers, with vertex 0 denoting a depot, and $A = \{(i, j) : i, j \in V, i \neq j\}$ is an arc set. Also define on A a non-negative cost or distance matrix $C = (c_{ij})$. With every vertex $i > 0$ is associated a known non-negative demand ξ_i to be delivered by a vehicle. A fleet of m identical vehicles of capacity Q is based at the depot. The VRP consists of designing a set of m vehicle routes whose cost is minimized in such a way that (i) every route starts and ends at the depot; (ii) every remaining city is visited exactly once by exactly one vehicle; (iii) the sum of demands on any vehicle route does not exceed Q . The VRP and several of its variants have been extensively studied in the Operations Research literature. For a recent survey, see Laporte (1992).

In the *Stochastic Vehicle Routing Problem* (SVRP), demands ξ_i are random variables. In general, the ξ_i 's need not be independent, identically distributed (i.i.d.) random variables, though this is assumed in this paper. Delivery routes are planned in a first stage and the individual demands are revealed at later stages upon a first customer visit by a vehicle. As a result, planned routes must account for the possibility that vehicle capacity may be exceeded by the actual demands at some point(s) on the route. When this happens, a *failure* is said to occur. In principle, each time a customer's demand is revealed, a decision has to be taken as to where the vehicle should proceed. This would lead to a stochastic dynamic formulation of the SVRP which is intractable for all but trivial size problems (see Dror, Laporte, and Trudeau, 1989; Dror, 1992). In this paper we examine the SVRP in the context of Stochastic Programming.

Stochastic Programming can be viewed as an extension of linear and non-

linear programming models which require a 'here-and-now' decision without making further observations of the random variables which, in our context, correspond to the customers' demands. In Stochastic Programming, it is common to distinguish between *Chance Constrained Programming* (CCP) and *Stochastic Programming with Recourse* (SPR). The objective in Stochastic Programming is to find an *a priori* solution which minimizes the expected cost over all potential realizations of the random variables. For the SVRP with recourse this implies finding a routing sequence together with a recourse policy which minimizes the expected distance for the fleet of the delivery vehicles. For a recent survey of Stochastic Programming, see Wets (1989).

The CCP versions of the SVRP usually consist of minimizing the sum of planned route costs, while ensuring that the probability of route failure does not exceed a given threshold ϵ . Stewart and Golden (1983) and Laporte, Louveaux, and Mercure (1989) have shown that under mild assumptions, chance constrained SVRPs reduce to deterministic VRPs. Their solution does not, however, provide information on the cost of recourse action in case of route failure.

In the SVRP with recourse, the problem is to plan m vehicle routes starting and ending at the depot, in order to minimize an objective function consisting of the sum of planned route costs, plus the expected cost of recourse actions, which may take different forms, some restricted such as returning to the depot to reload and resume deliveries along the planned route, or visiting the remaining customers possibly in an order that differs from the planned sequence. Clearly the recourse action may include *breaks* at some points along the route which can be planned *a priori* (restricted recourse) (Dror, Laporte, and Trudeau, 1989; Laporte and Louveaux, 1990). The cost of the recourse action is defined as the

expected total cost of all return trips between customers and the depot due to planned breaks or failures, minus the expected value of any savings resulting from customer resequencing after a failure. The SVRP with recourse is considerably more difficult to solve than the chance constrained SVRP. Integer programming formulations have been proposed (Dror, 1992, and Laporte and Louveaux, 1990) but so far, no exact algorithms have ever been devised and implemented for the general case. In this paper we examine the SVRP in the context of Chance Constrained Stochastic Programming and Stochastic Programming with recourse where a recourse action is determined upon failure or in anticipation of failure.

Even if no optimal algorithm can realistically be set up for general SVRPs, we believe that large class of problems of this kind can be tackled by appropriately exploiting solution methodologies originally developed for related deterministic problems. The purpose of this paper is to identify such a class of problems and to suggest solution approaches. In particular, we will examine the chance constrained SVRP and a class of SVRPs with recourse (to be more specifically defined later) in the context where there is only one vehicle and the number of potential failures is small. There are several situations where this is the case, as it makes little managerial sense to plan distribution routes with a large number of built-in failures. Failures, if they occur at all, are generally few and often tend to happen towards the end of the route. More precisely, given a set of customers, say $\{1, 2, \dots, n\}$, on a single route, the probability of a route failure at the k^{th} customer on the route and no route failure occurred before is $P(\sum_{i=1}^{k-1} \xi_i \leq Q < \sum_{i=1}^k \xi_i)$. In Kreimer and Dror (1990), it is proven for a number of common probability distributions, that given the ξ_i 's are i.i.d. random variables with coefficient of variation not exceeding one, the

probability of failure is monotonically increasing in the range $1, \dots, k^*$, where $k^* = \max_k \{\mu k\} \leq Q, \mu = E[\xi_i]$. It is also conjectured that this monotonicity property holds for a much larger class of probability distributions and in principle, the random variables need not to be identically distributed (see also Keilson, 1971). It is reasonable to assume that the mean total demand of customers on a route rarely very much exceeds the capacity of a vehicle. This assumption together with the results from Kreimer and Dror (1990), essentially imply that the number of potential route failures is small, and if a failure happens, it occurs at one of the last customers on the route.

We will show that problems belonging to this class can often be solved to optimality by making appropriate use of algorithms already developed for the deterministic VRP or for the *Traveling Salesman Problem* (TSP). One should keep in mind the vast research body developed over the years for solving hard deterministic combinatorial optimization problems such as the TSP and the VRP, and compare it to the limited research into general stochastic programming models for routing problems. Here, we attempt to exploit some of the existing solution methodologies developed for deterministic combinatorial problems in order to solve a class of stochastic combinatorial problems.

The remainder of the paper is organized as follows. The chance constrained SVRP with restricted failures is examined in Section 1. In Section 2, three versions of the SVRP with recourse are considered, and the conclusion follows in Section 3.

1. The Chance Constrained SVRP with Restricted Failures

We assume that the random variables ξ_i are such that $P(\xi_i > Q) = 0$ for all i and that $P(\sum_{i \in T} \xi_i > a)$ can be efficiently computed for any real number a

and for any non-empty subset T of V . For example, this is the case for several common distributions (the Poisson, Normal, and Gamma distributions) given that the individual demands are independent and identically distributed.

Define $\text{SVRP}(k)$ as the chance constrained problem consisting of designing m vehicle routes of least total cost, subject to constraints (i), (ii) of the VRP and (iv): the probability of having more than k failures on any route is negligible (say less than some $\epsilon > 0$). Let z_k be the optimal solution value of $\text{SVRP}(k)$. Note that this value does not correspond to the expected solution value since the expected cost of failure is not accounted for. The problem can be solved as a standard VRP in which routes with more than k potential failures are eliminated.

Let x_{ij} be a binary variable equal to 1 if and only if arc (i, j) appears in the optimal solution. The formulation of $\text{SVRP}(k)$ is then

$$\text{minimize } \sum_{i \neq j} c_{ij} x_{ij} \quad (1)$$

subject to

$$\sum_{i=1}^n x_{i0} = \sum_{j=1}^n x_{0j} = m \quad (2)$$

$$\sum_{i \neq l} x_{il} = \sum_{j \neq l} x_{lj} = 1 \quad (l \in V \setminus \{0\}) \quad (3)$$

$$\sum_{i \notin T, j \in T} x_{ij} \geq v(T, k) \quad (T \subset V \setminus \{0\}, 2 \leq |T| \leq n-1) \quad (4)$$

$$x_{ij} \in \{0, 1\} \quad ((i, j) \in A) \quad (5)$$

In this formulation, constraints (2), (3), and (5) are self-explanatory. In connectivity constraints (4), $v(T, k)$ is the minimum number of vehicles entering T so that the probability of there being more than k failures in T is negligible. If the demands are discrete, computing $v(T, k)$ exactly requires the solution of bin

packing problems. To circumvent this difficulty, one can first generate routes using a lower bound $\underline{v}(T, k)$ on $v(T, k)$, and check for feasibility only in the final set of routes, then reiterate if violations of constraints (4) are found. As in the case of the TSP, it is always valid to set $\underline{v}(T, k) = 1$, which corresponds to connectivity constraints. However, a stronger value of 2 can be used whenever T cannot feasibly be covered with one vehicle, even if split deliveries are allowed, i.e., when $P(\sum_{i \in T} \xi_i > kQ) \geq \epsilon$. These constraints are a direct extension of similar constraints used in the VRP (Laporte, Nobert, and Desrochers, 1985) and in a more general version of the chance constrained SVRP (Laporte, Louveaux, and Mercure, 1989).

It is simple to extend this model to require no more than h ($\geq k$) failures on all the routes combined in addition to limiting the number of failures on any single route to at most k . The formulation is identical to (1) - (5) but for the value m in constraint (2) which is set to satisfy $m \geq v(V \setminus \{0\}, h)$.

If C is asymmetric, the problem (1) - (5) can be solved as in Laporte, Mercure, and Nobert (1986), by adapting the Carpaneto and Toth (1980) algorithm (a number of other known TSP algorithms could be used just as well), for example :

Step 1: Transform the unconstrained VRP into a TSP by using the graph extension proposed by Lenstra and Rinnooy Kan (1975).

Step 2: Apply to the TSP the Carpaneto and Toth algorithm, by considering as illegal not only subtours disconnected from $\{0\}$, but also all paths corresponding to subsets T violating constraints (4).

Step 3: Interpret the TSP on the extended graph as a VRP solution on the original graph. $\square$

If C is symmetric, x_{ij} needs only to be defined for $i < j$ and it may be more efficient to resort to Integer Linear Programming, as in Laporte, Louveaux, and Mercure (1989). Another possibility is to use a polytopal cutting plane algorithm (Padberg and Rinaldi, 1990, Grötschel and Holland, 1991) where, in addition to subtour elimination constraints (4), cuts such as 2-matching inequalities, comb inequalities, and clique tree inequalities are generated.

It should be apparent that solving this kind of integer programming model which incorporates uncertainties about the customers' real demands through subtour elimination constraints (4) is comparable to solving a deterministic VRP. In this case, it is of interest to compare, for a given $\epsilon > 0$ in the CCP problem, the value of z_k to the expected value of the corresponding recourse model since z_k does not include the expected cost of the recourse.

Given that we are interested in solutions to real-life problems, it is safe to assume that no delivery routes with numerous failures have to be considered. Thus, we can use model (1) - (5) to construct initial routes with $1 - \epsilon$ probability of having at most one failure per route, by setting $k = 1$ in $v(T, k)$. We can now proceed to the next section in which a class of Stochastic Programming with recourse models is solved for the case where the probability of having more than one failure per route is negligible.

2. The SVRP with Recourse for a Maximum of One Failure per Route

In a number of practical applications like the delivery of home heating oil, the demand distribution is such that only one failure per route can occur, at one of the last k vertices of a route (Assad, Golden, Dahl, and Dror, 1983). This is not, as in the previous section, a constraint imposed on the problem structure, but a characteristic of the input data. We present an analysis of this problem

for the case $m = 1$, since as a first phase, we can solve the CCP version of the SVRP and obtain routes with no more than one likely failure each.

For this basic case we examine a number of different recourse policies: (1) assume that customers are visited sequentially on the planned route until a failure occurs (if no failure occurs, the route is followed to completion as planned). In such a case, the vehicle returns to the depot to reload and resumes its deliveries as planned from the vertex where failure occurred; (2) customers are visited sequentially on the planned route until a failure occurs, at which point the vehicle returns to the depot to reload and optimizes its remaining deliveries; and (3) breaks are planned along the vehicle route; upon failure, a return trip to the depot is made and the deliveries are resumed as planned.

Recourse Policy 1: no reoptimization

Here we assume that if a failure occurs, this happens at one of the last k customers on the route. For $k > 1$, let $V_k^l = \{i_1, \dots, i_{k-1}, 0\}$ be the l^{th} lexicographically ordered subset of cardinality k of V ending with vertex 0; if $k = 1$ define V_k^l as $\{0\}$. Denote by i^* the first element of V_k^l (if $k > 1$, then $i^* = i_1$; if $k = 1$, then $i^* = 0$). Let z_k^l be the cost of an optimal SVRP solution in which the last vertices correspond to the ordered sequence of the elements of V_k^l , and failure occurs either in V_k^l , or at the vertex immediately preceding i^* on the tour. The value of z_k^l is obtained by adding

- a) the cost of an optimal Hamiltonian path starting at 0 and ending at i^* , containing all elements of $V \setminus V_k^l$;
- b) if $k > 1$, the cost $c_{i_{k-1},0} + \sum_{i=1}^{k-2} c_{i,i+1}$ of the sequence $(i_1, \dots, i_{k-1}, 0)$, where the summation is zero when $k = 2$;
- c) if $k > 1$, the expected cost of recourse in V_k^l : i.e., $\sum_{i=1}^{k-1} (c_{i,0} + c_{0i_i}) p_i$.

where p_t is the probability of failure at vertex i_t and is computed as

$$p_t = P\left(\sum_{i \in V \setminus \{i_t, \dots, i_{k-1}\}} \xi_i \leq Q < \sum_{i \in V \setminus \{i_{t+1}, \dots, i_{k-1}\}} \xi_i\right) \quad (6)$$

with $\{i_{t+1}, \dots, i_{k-1}\} = \emptyset$ if $t = k - 1$.

Recourse Policy 2: reoptimization of the customer sequence after failure

If we request that the customer sequence after a failure be reoptimized, then an approach similar to the one described for Policy 1 can be used. Define V_k^t as the t^{th} lexicographically ordered subset of cardinality *not exceeding* k and ending with vertex 0, and compute z_k^t as the sum of two terms:

a) the first term is defined as in Policy 1, and;

b) if $k > 1$, the expected cost of the remaining sequence starting at i^* : $\sum_{t=1}^{k-1} p_t [(c_{i_t,0} + c_{0,i_t}) + R^*(i_t)]$, where p_t is defined by (6), and $R^*(i_t)$ is the cost of the optimal Hamiltonian path starting at i_t , ending at 0, and including all vertices of $\{i_{t+1}, \dots, i_{k-1}\}$ (this set is empty if $t = k - 1$).

The procedure just described requires the solution of $O(n^{k-1})$ TSPs for Policy 1, and of $O(kn^{k-1})$ TSPs for Policy 2, and is therefore impractical for large values of n and k . It is, however, attractive in a number of contexts where failure can only occur at one of the last few customers of the route. When $k > 1$, the number of TSP solutions required can be substantially reduced by means of an appropriate enumerative procedure that uses strong TSP lower bounds to fathom subproblems (see, e.g., Balas and Toth, 1985).

Finally, when the demand distributions are such that failure may occur before the last k customers of the route, our results can still be used to derive an SVRP solution (optimal for a given k) and then compute the probability of it being optimal in the given setting.

Proposition 1. For a given k , the probability that the above procedure will yield an optimal SVRP solution is greater or equal to $1 - \max_i P(\sum_{i \in V \setminus V_k^i} \xi_i > Q)$. $\square$

To illustrate the usefulness of this result, consider recourse Policy 1 and let $n = 10, Q = 100$ and $\xi_i \sim N(10, 4)$ for all i . Then,

$$P(\sum_{i \in V \setminus V_k^i} \xi_i > Q) = \begin{cases} 0.0475 & \text{if } k = 1 \\ 0.0002 & \text{if } k = 2 \\ 0.0000 & \text{if } k \geq 3. \end{cases}$$

In other words, solving a single TSP yields an SVRP solution where the probability of being optimal exceeds 95%; if $O(n)$ TSPs are solved, it is almost certain that the solution will be optimal. The value of k can be selected taking into account the probability of not obtaining the optimal SVRP solution in a given problem setting. Increasing k and solving either recourse policy version 1 or 2 will increase the probability of obtaining an optimal solution. The associated computational burden would then have to be weighted against the probability of obtaining a suboptimal solution.

Recourse Policy 3: planning a preventive break or risking failure

In the following, we develop a model that allows choosing between planning a preventive break along the route, or risking a failure. The full mathematical model for this case is very involved (see Laporte and Louveaux, 1990) and computationally intractable. However, an interesting case occurs when failure can only happen once and at the last visited vertex, i.e., $P(\sum_{i \in V \setminus \{j\}} \xi_i \leq Q) = 1$ for all $j \in V$. Here, the probability of route failure is a constant equal to $p = P(\sum_{i \in V} \xi_i > Q)$.

One possible policy is to risk a failure at the last vertex and then incur a penalty equal to the expected length of the return trip between the last customer on the route and the depot. This can be easily handled by modeling the SVRP as a TSP in which all costs c_{i0} are replaced by $c_{i0} + p(c_{i0} + c_{0i})$. Another possibility is to plan a break at a suitable location along the route and avoid potential route failure. Such an action may be viewed as an "insurance policy" against failure. This too can be formulated as a TSP by introducing a replica of the depot in the set of vertices. Thus, by solving two deterministic TSPs, it is possible to select the best policy. The optimal course of action can, however, be determined by solving a single TSP over $|V| + 2$ vertices by means of a suitable transformation. To explain the transformation, we proceed in two steps.

First, introduce two artificial vertices $n+1$ and $n+2$ with the following interpretations: if $n+1$ is entered from a vertex of $V \setminus \{0\}$, this indicates the solution includes a preventive break; if $n+2$ is entered from a vertex of $V \setminus \{0\}$, the solution contains no preventive break, but there is a risk of incurring the cost of a return trip between the last visited customer and the depot if a failure occurs. In this first transformation, only one of the vertices $n+1$ or $n+2$ will be visited. The following costs are associated with vertices $n+1$ and $n+2$ vertices:

$$c_{i,n+1} = c_{i0} \quad (i \in V \setminus \{0\})$$

$$c_{n+1,j} = c_{0j} \quad (j \in V \setminus \{0\})$$

$$c_{0,n+1} = c_{n+1,0} = \infty$$

$$c_{i,n+2} = c_{i0} + p(c_{i0} + c_{0i}) \quad (i \in V \setminus \{0\})$$

$$c_{n+2,j} = \infty \quad (j \in V \setminus \{0\})$$

$$c_{0,n+2} = \infty$$

$$c_{n+2,0} = 0$$

$$c_{n+1,n+2} = c_{n+2,n+1} = \infty$$

Define $S_i = \{i\}$ for $i \in V$ and $S_{n+1} = \{n+1, n+2\}$. Then the problem is to determine a least cost Hamiltonian circuit passing through each of the sets $S_0, S_1, \dots, S_{n+1}$ exactly once. This is an asymmetric *Generalized Traveling Salesman Problem* (GTSP) for which solution methodologies are now well known (see, e.g., Noon and Bean, 1991).

The second step consists of transforming the GTSP into a TSP over the same number of vertices as there are sets. Here, we proceed as in Noon and Bean (1992). First, redefine $c_{n+1,n+2} = c_{n+2,n+1} = -\lambda$, where λ is an arbitrarily large positive number, thereby forcing the two vertices of S_{n+1} to be visited consecutively; then interchange the costs of going out of the vertices $n+1$ and $n+2$, i.e., redefine

$$c_{n+1,j} = \infty \quad (j \in V \setminus \{0\})$$

$$c_{n+1,0} = 0$$

$$c_{n+2,j} = c_{0j} \quad (j \in V \setminus \{0\})$$

$$c_{n+2,0} = \infty$$

Therefore, when failure can occur only at the last node, an optimal solution to the SVRP with recourse that includes a possible preventive break can be obtained by solving a single TSP over $|V| + 2$ vertices.

3. Conclusion

The SVRP is notoriously difficult to solve to optimality, even under the most simple recourse policies. In this paper, we have shown how some classes of the

problem can be handled by making appropriate use of methodology already developed for the deterministic capacitated VRP, and for the TSP. These are problems for which the number of failures is restricted and often occur for the customers whose planned visits are last. This class of problems is realistic since it makes little sense to plan delivery routes that are likely to fail a large number of times.

We have shown how chance constrained SVRPs with at most k failures per route can be formulated and solved almost like the capacitated VRP. Thus, solving these problems to optimality for small values of k poses no additional difficulty. The chance constrained SVRP formulation does not, however, account for the expected cost of failure. We have considered three versions of the SVRP with recourse for which failures are known to occur at one of the last k customers of the planned route, and in which only one vehicle is used. In the first two scenarios, the problem can be solved to optimality by solving in a branch and bound manner $O(kn^{k-1})$ or $O(n^{k-1})$ TSPs, sometimes only one. This is computationally feasible since in practice k is small, and with today's algorithms, the TSP can be solved exactly in short computing times for relatively large values of n (see, e.g., Grötschel and Holland (1991) and Miller and Pekny (1991)). Therefore, with our approach, an interesting class of SVRPs with recourse can be solved to optimality. If failure is not known to occur at one of the last k customers, this type of methodology can still be used to obtain a feasible solution and compute *a priori* a lower bound on the probability of it being optimal. Under the last scenario considered, failure can only happen once at the last customer of the planned route. In this case, the SVRP with recourse can be solved to optimality as a TSP with only two extra vertices.

Acknowledgements

The first two authors were partially supported by the Canadian Natural Sciences and Engineering Research Council, grants OGP0036429 and OGP0039682.

Thanks are due to the referees for their valuable comments.

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